# Circles

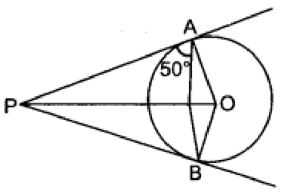
## 2016

### Very Short Answer Type Questions [1 Mark]

**Question 1.**

From an external point P, tangents PA and PB are drawn to a circle with centre O. If ∠PAB = 50°, then find ∠AOB.

**Solution:**




Given, $\angle PAB = 50^\circ$

$\angle PAB + \angle OAB = 90^\circ$

[∵ angle between radius OA and tangent PA is 90°]

$\Rightarrow \quad 50^\circ + \angle OAB = 90^\circ$

$\Rightarrow \quad \angle OAB = 90^\circ - 50^\circ = 40^\circ$

Now, $PA = PB$ [∴ tangents from an external point are same]

$\Rightarrow \quad \angle PBA = \angle PAB$

$\Rightarrow \quad \angle PBA = 50^\circ$

$\angle PBA + \angle OBA = 90^\circ$ [∴ angle between radius OB and tangent PB is 180°]

$\Rightarrow \quad 50^\circ + \angle OBA = 90^\circ$

$\Rightarrow \quad \angle OBA = 90^\circ - 50^\circ = 40^\circ$

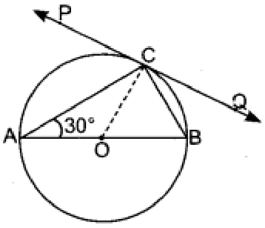
Now in $\Delta AOB$ we have

$\angle AOB + \angle ABO + \angle BAO = 180^\circ$ [∵ sum of angles in triangle is 180°]

$\Rightarrow \quad \angle AOB + 40^\circ + 40^\circ = 180^\circ \Rightarrow \angle AOB = 180^\circ - 80^\circ = 100^\circ$

**Question 2.**

In given figure, PQ is a tangent at a point C to a circle with centre O. If AB is a diameter and ∠CAB = 30°, find ∠PCA




**Solution:**

**Construction:** Join AO.

**Given:** PQ is tangent. AB is diameter $\angle CAB = 30^\circ$.

**To Find:** $\angle PCA$

**Solution:** In $\Delta AOC$, $AO = CO$ (∵ Equal radii)

$\angle CAO = \angle OCA$ (∵ Angles opposite to equal sides are equal)

or $\angle CAB = \angle OCA$

But, $\angle CAB = 30^\circ$ So, $\angle OCA = 30^\circ$ ... (*i*)

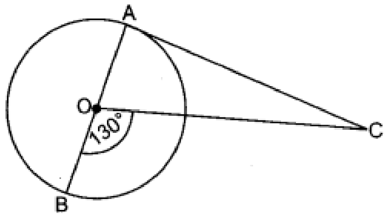
Since, $OC \perp PQ$ (∵ Tangent is perpendicular to radius at point of contact)

$\Rightarrow \quad \angle PCO = 90^\circ \Rightarrow \angle OCA + \angle PCA = 90^\circ \Rightarrow 30^\circ + \angle PCA = 90^\circ$

$\therefore \quad \angle PCA = 60^\circ$

**Question 3.**

In figure given, AOB is a diameter of a circle with centre O and AC is a tangent to the circle at A. If ∠BOC = 130°, then find ∠ACO.




**Solution:**

$$\angle AOC + \angle BOC = 180° \quad [\because \text{Linear Pair Axiom}]$$
$$\angle AOC + 130° = 180°$$
$$\angle AOC = 180° - 130°$$
$$\angle AOC = 50°$$

Now, $\angle OAC = 90°$ [angle between radius OA and tangent AC is 90°]

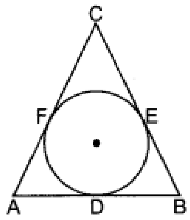
Now, in $\Delta AOC$,

$$\angle OAC + \angle AOC + \angle ACO = 180° \quad [\because \text{sum of angles in triangle is } 180°]$$
$$90° + 50° + \angle ACO = 180°$$
$$\angle ACO = 180° - 140°$$
$$\angle ACO = 40°$$

## Short Answer Type Questions I [2Marks]

**Question 4.**

In given figure, a circle is inscribed in a ΔABC, such that it touches the sides AB, BC and CA at points D, E and F respectively. If the lengths of sides AB, BC and CA are 12 cm, 8 cm and 10 cm respectively, find the lengths of AD, BE and CF




**Solution:**

Given, AB = 12 cm, CA = 10 cm, BC = 8 cm

Let $AD = AF = x$ [$\because$ Tangent drawn from external point to circle are equal]

$\therefore \quad DB = BE = 12 - x$ and $CF = CE = 10 - x$

$BC = BE + EC \Rightarrow 8 = 12 - x + 10 - x$

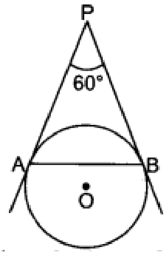
$\Rightarrow \quad x = 7$

$\therefore$ AD = 7 cm, BE = 5 cm and CF = 3 cm

**Question 5.**

If given figure, AP and BP are tangents to a circle with centre O, such that AP = 5 cm and

$\angle APB = 60°$. Find the length of chord AB.




**Solution:**

In $\Delta APB$ we have $AP = BP$

$\Rightarrow \angle PAB = \angle PBA$

[$\because$ Tangents from an external point are equally inclined to segment joining centre to point]

Let $\angle PAB = x$,

then in $\Delta APB$, $x + x + 60° = 180°$

$2x = 180° - 60° = 120°$

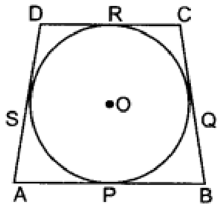
$x = 60°$

As all three angles of $\Delta APB$ are 60°. So $\Delta APB$ is an equilateral triangle.

Hence $AP = BP = AB = 5$ cm

**Question 6.**

In figure, a quadrilateral ABCD is drawn to circumscribe a circle, with centre O, in such a way that the sides AB, BC, CD and DA touch the circle at the points P, Q, R and S respectively. Prove that AB + CD = BC + DA.




**Solution:**

We know that tangents drawn to a circle from an outer points are equal.

So, $AP = AS$, $BP = BQ$,

$CR = CQ$ and $DR = DS$.

Now, consider

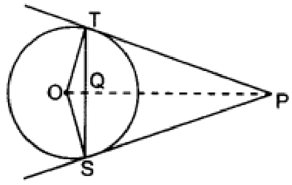
$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

$$\Rightarrow AB + CD = AD + BC$$

Hence proved.

**Question 7.**

In given figure, from an external point P, two tangents PT and PS are drawn to a circle with centre O and radius r.If PO = 2r, show that $\angle OTS = \angle OST = 30°$.



**Solution:**

Let $\angle TOP = \theta$

In right triangle OTP we have

$$\therefore \quad \cos\theta = \frac{OT}{OP} = \frac{r}{2r} = \frac{1}{2} = \cos 60° \Rightarrow \theta = 60°$$

Hence $\angle TOS = 2 \times 60 = 120°$ [$\because$ $\angle TOP = \angle POS$ as angles opposite to equal tangent are equal]

In $\Delta OTS$, we have $OT = OS$ [$\because$ Equal radii]

$\Rightarrow \quad \angle OTS = \angle OST$ [$\because$ Angle opposite to equal sides are equal]

In $\Delta OTS$,

$$\angle OTS + \angle OST + \angle TOS = 180°$$

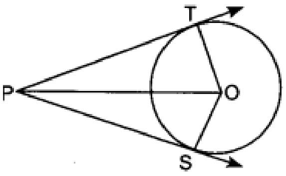
$$2\angle OST = 60°$$

$$\therefore \quad \angle OST = \angle OTS = 30°$$

Hence proved.

**Question 8.**

In given figure, from a point P, two tangents PT and PS are drawn to a circle with centre O such that ∠SPT = 120°, Prove that OP = 2PS




**Solution:**

Let $PT = x = PS$ [$\because$ Tangent drawn from external point to circle are equal]

$\angle SPT = 120°$

In $\Delta OTP$ and $\Delta OSP$, $\angle OTP = \angle OSP$

[$\because$ each equal to 90°, since tangent perpendicular $r$ radius]

$OT = OS$ [$\because$ Equal radii]

$OP = OP$ [common]

$\Rightarrow \quad \Delta OSP \cong \Delta OTP$ [$\because$ By SAS congruence rule]

$\therefore \quad \angle TPO = \angle SPO$ [$\because$ By CPCT]

$$\Rightarrow \quad \angle TPO = \frac{1}{2}\angle SPT = \frac{1}{2} \times 120 = 60°$$

In $\Delta OTP$, $\frac{OP}{x} = \text{Sec } 60°$

$$\Rightarrow \quad \frac{OP}{x} = 2 \Rightarrow OP = 2x \Rightarrow OP = 2PS$$

Hence proved.

**Question 9.**

In given figure, there are two concentric circles of radii 6 cm and 4 cm with centre O. If AP is a tangent to the larger circle and BP to the smaller circle and length of AP is 8cm, find the length of BP

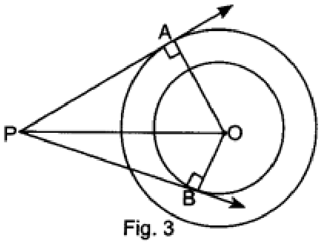


Fig. 3

**Solution:**

$OA = 6$ cm [$\because$ Given radius]

$OB = 4$ cm [$\because$ Given radius]

$AP = 8$ cm

In $\Delta OAP$, $OP^2 = OA^2 + AP^2 = 36 + 64 = 100$ [$\because$ Pythagoras theorem]

$\Rightarrow \quad OP = 10$ cm

In $\Delta OBP$, $BP^2 = OP^2 - OB^2 = 100 - 16 = 84$ [$\because$ Pythagoras theorem]

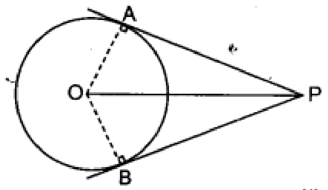
$BP = 2\sqrt{21}$ cm

## Long Answer Type Questions [4 Marks]

### Question 10.

Prove that the lengths of tangents drawn from an external point to a circle are equal

**Solution:**




**Given:** A circle $C(O, r)$, P is a point outside the circle and PA and PB are tangents to a circle.

**To Prove:** PA = PB

**Construction:** Draw OA, OB and OP.

**Proof:** Consider triangles OAP and OBP.

$$\angle OAP = \angle OBP = 90^\circ \quad \ldots(i)$$

[Radius is perpendicular to the tangent at the point of contact]

$$OA = OB \text{ (radii)} \quad \ldots(ii)$$

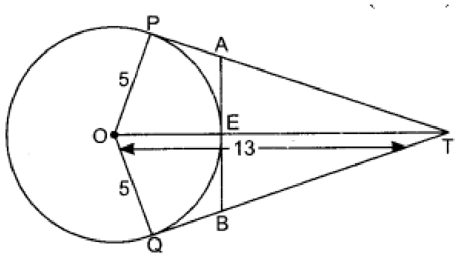
OP is common ...(iii)

$\therefore \quad \Delta OAP \cong \Delta OBP$ (RHS) [from (i), (ii) and (iii)]

Hence, $AP = BP$ (CPCT)

### Question 11.

**In given figure, O is the centre of a circle of radius 5 cm. T is a point such that OT = 13 cm and OT intersects circle at E. If AB is a tangent to the circle at E, find the length of AB, where TP and TQ are two tangents to the circle.**




**Solution:**

In $\Delta OPT$, $OP^2 + PT^2 = OT^2$ [$\because$ Pythagoras theorem]

$$PT = \sqrt{OT^2 - OP^2}$$

$$= \sqrt{169 - 25} = 12 \text{ cm}$$

and $TE = OT - OE = 13 - 5 = 8$ cm

Let $PA = AE = x$ [tangent from outer point A]

In $\Delta TEA$, $TE^2 + EA^2 = TA^2$ [$\because$ Pythagoras theorem]

$$(8)^2 + (x)^2 = (12 - x)^2$$

$$64 + x^2 = (12 - x)^2$$

$\Rightarrow \quad 64 + x^2 = 144 + x^2 - 24x$

$\Rightarrow \quad 80 = 24x \quad \Rightarrow x = 3.3$ cm

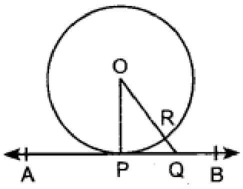
Thus $AB = 2 \times 3.3$ cm $= 6.6$ cm [$\because$ AE = EB, as AB is tangent to circle at E]

### Question 12.

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

**Solution:**

**Given:** A circle $C(O, r)$ and a tangent AB at a point P.
**To prove:** $OP \perp AB$
**Construction:** Take any point Q other than P on the tangent AB.
Join OQ, intersecting circle at R.




**Proof:** We have, $OP = OR$ [Radii]

$OQ = OR + RQ$

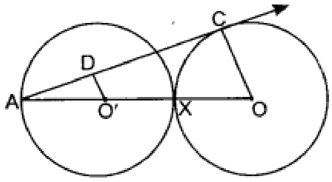
$\therefore$ $OQ > OR \Rightarrow OQ > OP$ [$\because$ OR = OP = radius]

Thus, $OP < OQ$, i.e. OP is shorter than any other segment joining O to any point of AB.
But among all line segments, joining point O to point on AB, shortest one is perpendicular from O on AB.

Hence, $OP \perp AB$

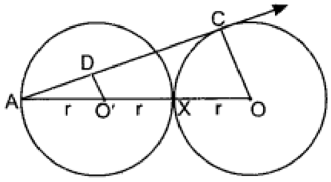
**Question 13.**

In given figure, two equal circles, with centres O and O', touch each other at X. OO' produced meets the circle with centre O' at A. AC is tangent to the circle with centre O, at the point C. O'D is perpendicular to AC. Find the value of DO'/CO.




**Solution:**

AC is tangent to the circle with centre O.




In $\Delta ADO'$ and $\Delta ACO$, $\angle ADO' = \angle ACO$ (each 90°)

$\angle DAO = \angle CAO$ (common)

$\therefore$ By AA criterion, $\frac{AO'}{AO} = \frac{DO'}{CO}$ [$\because$ corresponding parts of similar triangle]

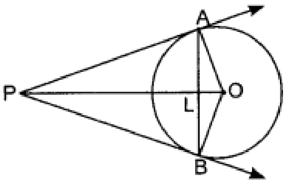
$AO = AO' + O'X + XO = r + r + r = 3r$

$\frac{DO'}{CO} = \frac{r}{3r}$ [$\therefore$ AO = AO' + O'X + XO = 3AO]

$\Rightarrow$ $\frac{DO'}{CO} = \frac{1}{3}$

**Question 14.**

In given figure, AB is a chord of a circle, with centre O, such that AB = 16 cm and radius of circle is 10 cm. Tangents at A and B intersect each other at P. Find the length of PA




**Solution:**

Let $\qquad PL = x$

As OP is perpendicular bisector of AB. Then

$$AL = BL = 8 \text{ cm}$$

In $\Delta ALO$, $\quad OL^2 = OA^2 - AL^2 = 10^2 - 8^2 = 36 \Rightarrow OL = 6$ cm

$$AP^2 = OP^2 - OA^2 \qquad [\because \text{ Pythagoras theorem}]$$

In $\Delta OAP$, $\quad AP^2 = (x+6)^2 - 10^2$

$$AP^2 = AL^2 + PL^2 \qquad [\because \text{ Pythagoras theorem}]$$

In $\Delta ALP$, $\quad AP^2 = x^2 + 64$

Now, $\quad (x+6)^2 - 10^2 = x^2 + 64$

$$x^2 + 12x + 36 - 100 = x^2 + 64$$

$\Rightarrow \quad 12x = 128$

$\Rightarrow \quad x = \dfrac{128}{12}$

$$= \frac{32}{3} \text{ cm}$$

From $\Delta ALP$, $\quad AP^2 = \left(\dfrac{32}{3}\right)^2 + 64$

$$= \frac{1024}{9} + 64$$

$$= \frac{1024 + 576}{9} \text{ cm}$$

$$AP^2 = \frac{1600}{9} \text{ cm}$$

$$AP = \frac{40}{3} \text{ cm} = 13.3 \text{ cm}$$

## 2015

### Very Short Answer Type Questions [1 Mark

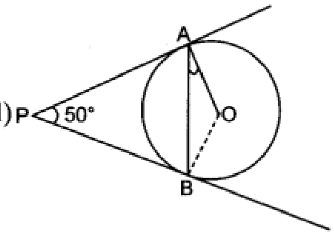
**Question 15.**

In figure, PA and PB are tangents to the circle with centre O such that ∠APB = 50°. Write the measure of ∠OAB

**Solution:**

Join OB.

$\because$ PA and PB are tangents to the circle drawn from an external point P. We know that, tangent is perpendicular r to radius.




$$\angle OAP = \angle OBP = 90°$$

Then, $\angle OAP + \angle APB + \angle OBP + \angle AOB = 360°$

(ASP of quadrilateral)

$\therefore \quad \angle APB + \angle AOB = 180°$

$\Rightarrow \quad 50° + \angle AOB = 180°$

$\Rightarrow \quad \angle AOB = 130°$

In $\Delta OAB$, $\quad OA = OB \qquad (\because$ equal radii)

$\Rightarrow \quad \angle A = \angle B = x$ (say)($\because$ angles opposite to equal sides are equal)

$\angle A + \angle B + \angle AOB = 180° \qquad (\because$ ASP of triangles)

$\Rightarrow \quad x + x + 130° = 180°$

$\Rightarrow \quad 2x = 50°$

$\Rightarrow \quad x = 25°$

$\therefore \quad \angle OAB = 25°$

**Question 16.**

Find the relation between x and y if the points A(x, y), B(-5, 7) and C(-4, 5) are collinear

**Solution:**

$\because$ A, B and C are collinear. Area of triangle $= \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)]$

$$\text{So,} \quad ar(\Delta ABC) = 0$$

$$\therefore \quad \frac{1}{2}[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)] = 0$$

$$\frac{1}{2}[x(7-5)+(-5)(5-y)+(-4)(y-7)] = 0$$

$$2x-25+5y-4y+28 = 0$$

$$\Rightarrow \quad 2x+y+3 = 0.$$ Required relation between $x$ and $y$.

**Question 17.**

Two concentric circles of radii a and b(a > b) are given. Find the length of the chord of the larger circle which touches the smaller circle.

**Solution:**

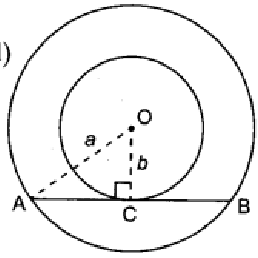
AB is tangent at C to circle C(O, $b$)

$$\therefore \quad OC \perp AB$$

$$\therefore \quad \angle OCB \perp 90^\circ$$

$$\Rightarrow \quad AC = BC \Rightarrow AB = 2AC$$

($\because$ perpendicular from centre to chord bisects chord)




Now, in $\Delta OCA$, $AO^2 = OC^2 + AC^2$

$$\Rightarrow \quad a^2 = b^2 + AC^2$$

$$\Rightarrow \quad AC = \sqrt{a^2-b^2}$$

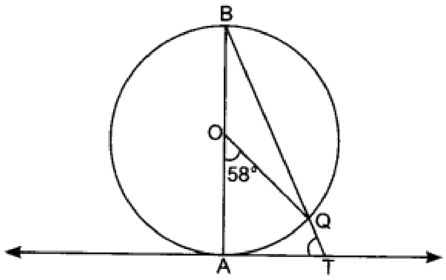
$$\therefore \quad AB = 2\sqrt{a^2-b^2} = 2AC$$

$$\text{length of chord} = 2\sqrt{a^2-b^2}$$

## Short Answer Type Questions I [2 Marks]

**Question 18.**

In figure, AB is the diameter of a circle with centre O and AT is a tangent. If ∠AOQ = 58°, find ∠ATQ




**Solution:**

$\because$ AT is a tangent and BA is a diameter.

$$\text{So,} \quad OA \perp AT$$

[radius is perpendicular to the tangent at point of contact]

$$\Rightarrow \quad \angle OAT = 90^\circ \quad \text{or} \quad \angle BAT = 90^\circ$$

Arc AQ subtends an angle of 58° at the circle.

$$\angle AOQ = 2\angle ABQ$$

$$\text{So,} \quad \angle ABQ = 29^\circ$$ [angle subtended by the arc at the centre is double the angle subtended by the same arc on the circle]

In $\Delta ABT$,

$$\angle A + \angle ABT + \angle ATB = 180^\circ$$

$$\Rightarrow \quad 90^\circ + 29^\circ + \angle ATB = 180^\circ$$

$$\Rightarrow \quad \angle ATB = 61^\circ$$

Hence, $\angle ATQ = 61^\circ$

**Question 19.**

From a point T outside a circle of centre O, tangents TP and TQ are drawn to the circle.

Prove that OT is the right bisector of the line segment PQ.

**Solution:**

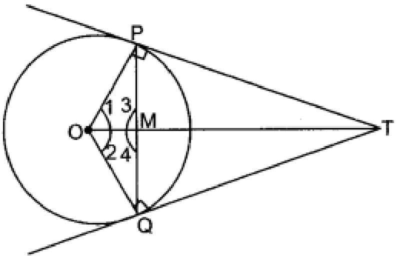
**Given:** TP and TQ are tangents to the circle of centre O.

**To Prove:** $\angle OMP = 90°$ and PM = MQ.

**Proof:** $\because$ TP and TQ are tangents at P and Q respectively.

So, $OP \perp PT$ and $OQ \perp QT$

($\because$ radius is perpendicular to the tangent at point of contact)




$\therefore \quad \angle OPT = \angle OQR = 90°$

In $\Delta OPT$ and $\Delta OQT$

$OP = OQ$ (radius)

$\angle P = \angle Q$ (each 90°)

$OT = OT$ (common)

So, $\Delta OPT \cong \Delta OQT$ (By RHS)

$\Rightarrow \angle 1 = \angle 2$ (By CPCT)

Now, In $\Delta OMP$ and $\Delta OMQ$,

$OP = OQ$ (radius)

$\angle 1 = \angle 2$ (Proved above)

$OM = OM$ (common)

So, $\Delta OMP \cong \Delta OMQ$ (By SAS)

$\Rightarrow PM = MQ$ and $\angle 3 = \angle 4$ (By CPCT)

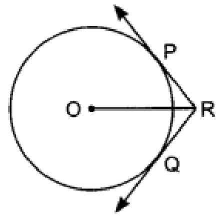
Now $\angle 3 + \angle 4 = 180°$ ($\because$ Linear Pair Axiom)

$\Rightarrow 2\angle 3 = 180° \Rightarrow \angle 3 = 90° \Rightarrow \angle OMP = 90°$

Hence, OT is the right bisector of the line segment PQ.

**Question 20.**

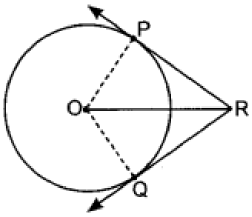
In figure, two tangents RQ and RP are drawn from an external point R to the circle with centre O. If $\angle PRQ = 120°$, then prove that OR = PR + RQ.




**Solution:**

We know that, tangent is perpendicular to radius. Perpendicular from centre bisects angle.

OR bisects $\angle PRQ$




$\therefore \quad \angle PRO = \angle QRO = 60°$

$[\because \angle PRQ = \angle ORP + \angle ORQ = 120°]$

In right $\Delta OPR$ ($\because OP \perp PR$) [$\because$ radius is perpendicular to the targent at point of contact]

$\therefore \quad \cos \angle ORP = \dfrac{PR}{OR} = \cos 60°$

$\Rightarrow \quad OR = 2PR \quad ...(i)$

Similarly, in right $\Delta OQR$, $\dfrac{QR}{OR} = \dfrac{1}{2} = \cos 60°$

$\Rightarrow \quad OR = 2QR \quad ...(ii)$

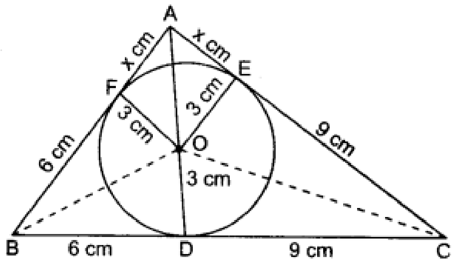
Adding (*i*) and (*ii*), we get

$2OR = 2PR + 2QR$

$\Rightarrow \quad OR = PR + RQ$

**Question 21.**

In figure, a triangle ABC is drawn to circumscribe a circle of radius 3 cm, such that the segments BD and DC are respectively of lengths 6 cm and 9 cm. If the area of $\Delta ABC$ is 54 cm², then find the lengths of sides AB and AC




**Solution:**

Let AF = $x$ cm, BC = (6 + 9) = 15 cm

$\because$ AF = AE [tangents drawn from an external point are equal]

$\therefore$ AE = $x$ cm

Also BD = BF = 6 cm

and CD = CE = 9 cm

$\therefore$ AB = $(x + 6)$ cm

In $\Delta$ABC, AC = $(x + 9)$ cm

$$\text{Area } \Delta ABC = \text{Area } \Delta BOC + \text{Area } \Delta COA + \text{Area } \Delta AOB$$

$$\Rightarrow \quad 54 = \frac{1}{2} BC \times OD + \frac{1}{2} AC \times OE + \frac{1}{2} AB \times OF$$

$$\Rightarrow \quad 54 \times 2 = 15 \times 3 + (9 + x) \times 3 + (6 + x) \times 3$$

$$108 = 45 + 18 + 3x + 27 + 3x$$

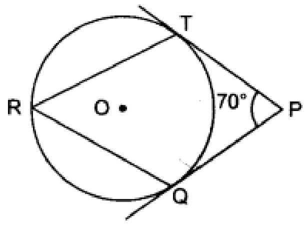
$$6x = 18 \Rightarrow x = 3$$

$$\Rightarrow \quad AB = 6 + x = 6 + 3 = 9 \text{ cm}$$

$$AC = 9 + x = 9 + 3 = 12 \text{ cm}$$

**Question 22.**

In figure, O is the centre of a circle. PT and PQ are tangents to the circle from an external point P. If ∠TPQ = 70°, find ∠TRQ




**Solution:**

We know that tangent is perpendicular to radius. Hence,

$$\angle OTP = \angle OQP = 90°$$

In quadrilateral PQOT,

$$\angle QOT + \angle OTP + \angle TPQ + \angle OQP = 360° \quad [\because \text{ ASP of quadrilateral}]$$

$$\angle TOQ + \angle TPQ = 180°$$

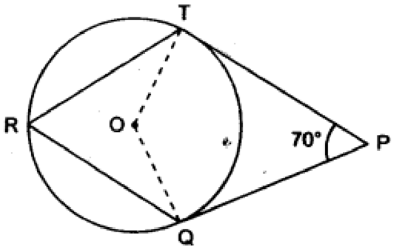
$$\Rightarrow \quad \angle TOQ = 110°$$

Also $\angle TOQ = 2\angle TRQ$

[angle subtended by an arc at centre of the circle is twice the angle subtended by it in alternate segment]

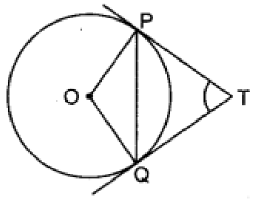
$$\Rightarrow \quad 110° = 2\angle TRQ$$

$$\Rightarrow \quad \angle TRQ = 55°$$



**Question 23.**

In figure, PQ is a chord of length 8 cm of a circle of radius 5 cm. The tangents at P and Q intersect at a point T. Find the lengths of TP and TQ




**Solution:**

Join OT intersecting PQ at R.

OT bisects $\angle PTQ$

$\therefore \quad \angle PTO = \angle QTO$

$\therefore \quad \angle PTR = \angle QTR \quad \ldots(i)$

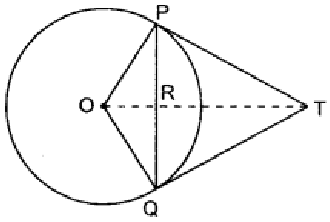
In $\Delta PTR$ and $\Delta QTR$, $PT = QT$ [length of tangents drawn from common external point are equal]

$RT = RT$ [common]

$\angle PTR = \angle QTR$ [$\because$ from $(i)$]

$\therefore \quad \Delta PTR \cong \Delta QTR$ [By SAS]

$\Rightarrow \quad PR = RQ$ [$\because$ By CPCT]




$\Rightarrow$ R is mid-point of PQ

$\therefore \quad OR \perp PQ$

In right triangle ORP

$OP^2 = PR^2 + OR^2$ [$\because$ Given, OP = 5 cm, PQ = 8 cm $\therefore$ PR = QR = 4 cm ]

$\Rightarrow \quad 25 = 16 + OR^2$

$OR = 3$ cm

In $\Delta ORQ$ and $\Delta OQT$

$\angle ORQ = \angle OQT$ (Each 90°)

$\angle ROQ = \angle ROQ$ (Common)

$\therefore \quad \Delta ORQ \sim \Delta OQT$ (By AA criterion)

$\Rightarrow \quad \dfrac{OR}{OQ} = \dfrac{RQ}{QT}$ (By C.P. of similar triangles)

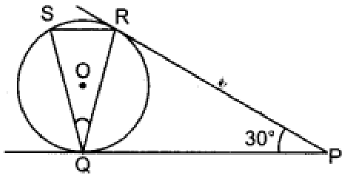
$\Rightarrow \quad \dfrac{3}{5} = \dfrac{4}{QT} \quad \Rightarrow \quad QT = \dfrac{20}{3}$ cm

Also $\quad PT = QT \quad \Rightarrow \quad PT = \dfrac{20}{3}$ cm

## Long Answer Type Questions [4 Marks]

**Question 24.**

In figure, tangents PQ and PR are drawn from an external point P to a circle with centre O, such that ∠RPQ = 30°. A chord RS is drawn parallel to the tangent PQ. Find ∠RQS




**Solution:**

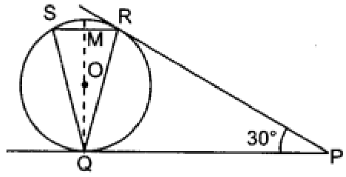
**Construction:** Draw a line through Q and perpendicular to PQ.

**Proof:** As $\quad MQ \perp PQ$

So, MQ passes through the centre O. [If a line is perpendicular to the tangent, then it must be passed through centre of the circle]

$\Rightarrow$ $\angle OQP = 90°$

Also $PQ = PR$ [lengths of tangents drawn from an external point to a circle are equal]




In $\Delta PQR$ $PQ = PR$

$\Rightarrow$ $\angle Q = \angle R$ [$\because$ angles opposite to equal sides are equal]

Now, $\angle P + \angle Q + \angle R = 180°$ [$\because$ ASP of triangle]

$\Rightarrow$ $30° + 2\angle Q = 180°$

$\Rightarrow$ $2\angle Q = 150°$

$\Rightarrow$ $\angle Q = 75°$

$\because$ Given, $SR \parallel PQ$

$\Rightarrow$ $\angle QRS = \angle RQP$ [if 2 lines are parallel, alternate angles are equal]

$\Rightarrow$ $\angle QRS = 75°$

Also, $\angle QSR = \angle RQP$ [$\because$ Corresponding angles equal]

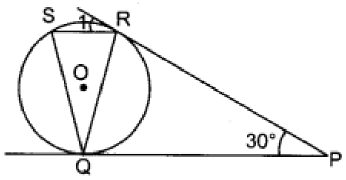
$\Rightarrow$ $\angle QSR = 75°$

Now, In $\Delta RQS$,

$\angle RQS + \angle RSQ + \angle SRQ = 180°$ [$\because$ ASP of triangle]

$\Rightarrow$ $\angle RQS + 75° + 75° = 180°$

$\Rightarrow$ $\angle RQS = 180 - 150 = 30°$




**Alternate method:**

**Given:** PQ and PR are tangents and $\angle RPQ = 30°$

**To find:** $\angle RQS$

$\because$ $SR \parallel QP$

$\Rightarrow$ $\angle 1 = \angle P$ [Corresponding angles are equal]

$\Rightarrow$ $\angle 1 = 30°$

$\because$ PR is tangent at P and $\angle 1 = 30°$

So, $\angle RQS = \angle 1$ [By alternate segment theorem]

$\Rightarrow$ $\angle RQS = 30°$

**Question 25.**

Prove that the tangent at any point of a circle is perpendicular to the radius throug the point of contact

**Solution:**

Refer to Ans. 12

**Question 26.**

Prove that the lengths of the tangents drawn from an external point to a circle are equal

**Solution:**

Refer to Ans. 10.

**Question 27.**

Prove that the tangent drawn at the mid-point of an arc of a circle is parallel to the chord joining the end points of the arc.

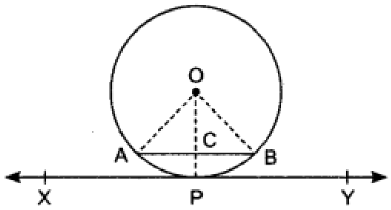
**Solution:**

**Given:** APB is arc of the circle $C(O, r)$, P is mid-point of arc APB and XY is tangent to the circle at P.

**To Prove:** AB | | XY

Join OA and OB

Here




$\angle AOP = \angle BOP$ (angle subtended by equal arc)

$OA = OB$ ($\because$ equal radii)

$OC = OC$ (common)

$\therefore$ $\Delta ACO \cong \Delta BCO$ (SAS)

So, $AC = BC$ (CPCT)

$\Rightarrow$ $OC \perp AB$

$\therefore$ $\angle OCB = \angle OCA = 90°$ (line joining mid-point of chord with centre of the circle is perpendicular to the chord)

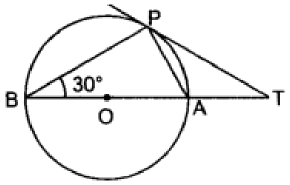
Also $\angle OPY = 90°$

$\Rightarrow$ $\angle OCB = \angle OPY$ (these are corresponding angles)

$\therefore$ AB | | XY ($\because$ if corresponding angles are equal, 2 lines are parallel)

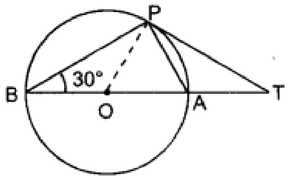
**Question 28.**

In figure, O is the centre of the circle and TP is the tangent to the circle from an external point T. If ∠PBT = 30°, prove that BA: AT = 2:1




**Solution:**

We have, $\angle BPA = 90°$ [$\because$ PT is tangent to circle, tangent perpendicular radius]




In $\Delta BPA$, $\angle ABP + \angle BPA + \angle PAB = 180°$ [$\because$ ASP of triangle]

$\Rightarrow$ $30° + 90° + \angle PAB = 180°$

[$\because$ $\angle PBT = \angle ABP = 30°$]

$\Rightarrow$ $\angle PAB = 60°$

Also $\angle POA = 2\angle PBA$

[$\because$ Angle subtended by an arc at centre is twice angle subtended by arc on circle]

$\Rightarrow$ $\angle POA = 2 \times 30° = 60°$

$\therefore$ $\angle PAO = \angle POA$

$\Rightarrow$ $OP = AP$ (sides opposite to equal angles) ...(*i*)

In $\Delta OPT$, $\angle OPT = 90°$ (radius is perpendicular to tangent)

$\angle POT = 60°$

$\therefore$ $\angle PTO = 30°$ [angle sum property of a triangle)

Also, $\angle APT + \angle ATP = \angle PAO$ (exterior angle property)

$\therefore$ $\angle APT + 30° = 60° \Rightarrow \angle APT = 30°$

$\angle PTA = \angle APT$ ($\because$ from above)

$\therefore$ $AP = AT$ (sides opposite to equal angles) ... (*ii*)

From (*i*) and (*ii*)

$\Rightarrow$ $AT = OP =$ radius of the circle $= r$ [$\because$ AP is radius of circle]

Now $AB = 2r$

$\Rightarrow$ $AB = 2AT \Rightarrow \dfrac{AB}{AT} = 2 \Rightarrow AB : AT = 2 : 1$

## 2014

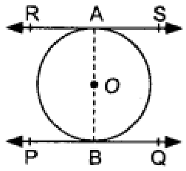
### Short Answer Type Questions I [2 Marks]

**Question 29.**

Prove that the line segment joining the points of contact of two parallel tangents of a circle, passes through its centre.

**Solution:**

**Given:** PQ and RS are two parallel tangents to a circle at B and A respectively. O is the centre of the circle. AOB be a line segment.




**To prove:** AB passes through O.

**Construction:** Join OA and OB.

**Proof:** As we know, OB is perpendicular to PQ.

[Tangent is perpendicular to radius at the point of contact.]

Now, given, PQ || RS ⇒ BO (Produced to RS) is perpendicular to RS. ....(*i*)

[A line perpendicular to one of the two parallel lines is perpendicular to other line also]

Also, OA is perpendicular to RS [∵ Tangent perpendicular to radius] ...(*ii*)

From (*i*) and (*ii*), OA and OB must coincide as only one line can be drawn perpendicular from a point outside the line to the line.

∴ AOB is straight line.

∴ A, O, B are collinear.

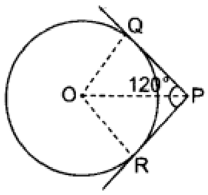
⇒ AB Passes through O, the centre of the circle.

**Question 30.**

If from an external point P of a circle with centre O, two tangents PQ and PR are drawn, such that ∠QPR = 120°, prove that 2PQ = PO.

**Solution:**

**Given:** PQ and PR are tangents from point P to circle with centre O.




Also, $\angle QPR = 120°$

**To Prove:** $2PQ = OP$

**Construction:** Join OQ, OP and OR

**Proof:** In triangles OQP and ORP,

$OQ = OR = r$ (say) [∵ equal radii]

$OP = OP$ (common)

$PQ = PR$

[The lengths of the tangents drawn from an external point to a circle are equal]

∴ $\Delta OQP \cong \Delta ORP$ (by SSS)

∴ $\angle OPQ = \angle OPR$ (By CPCT)

Now, given $\angle QPR = 120°$

⇒ $\angle OPQ + \angle OPR = 120°$

⇒ $2\angle OPQ = 120°$

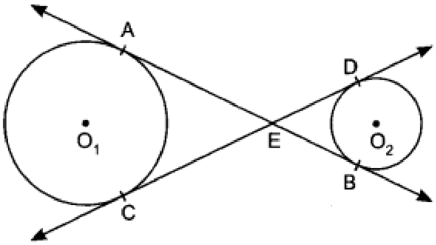
⇒ $\angle OPQ = 60° = \angle OPR$

**Now, In ΔOQP,** $\angle Q = 90°$ [∵ Tangent perpendicular to radius]

**Then,** $\dfrac{PQ}{OP} = \cos 60° = \dfrac{1}{2} \Rightarrow OP = 2PQ$

**Question 31.**

In figure, common tangents AB and CD to the two circles with Centres O1 and O2 intersect at E. Prove that AB = CD.




**Solution:**

In the given figure, AB and CD are common tangents to the two given circles with centres $O_1$ and $O_2$ respectively.

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

∴ $AE = EC$ and $EB = ED$

⇒ $AE + EB = CE + ED$

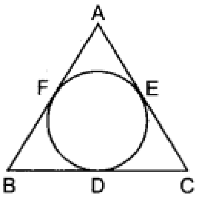
⇒ $AB = CD$.

**Question 32.**

The incircle of an isosceles triangle ABC, in which AB = AC, touches the sides BC, CA and AB at D, E and F respectively. Prove that BD = DC

**Solution:**

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.




$\therefore$ AF = AE, BF = BD and CD = CE ...(*i*)

Given AB = AC

$\Rightarrow$ AF + FB = AE + EC

$\Rightarrow$ FB = EC [using (*i*), AF = AE]

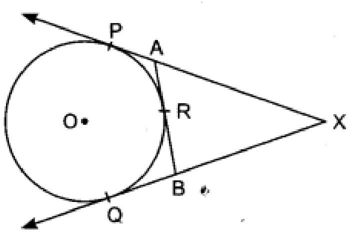
$\Rightarrow$ BD = CD [using (*i*), BF = BD and CD = CE]

**Question 33.**

In figure, XP and XQ are two tangents to the circle with centre O, drawn from an external point X. ARB is another tangent, touching the circle at R. Prove that XA+AR=XB+BR.

**Solution:**

Lengths of the tangents drawn from a point outside the circle to the circle are equal.




$\therefore$ XP = XQ, AP = AR and BR = BQ ...(*i*)

Now, XP = XQ [$\because$ equal tangents]

$\Rightarrow$ XA + AP = XB + BQ

$\Rightarrow$ XA + AR = XB + BR [using (*i*)]

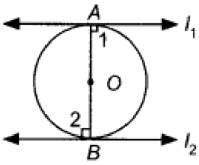
Hence proved.

**Question 34.**

Prove that the tangents drawn at the ends of any diameter of a circle are parallel.

**Solution:**

AB is diameter of a circle with centre O and $l_1$, $l_2$ are the tangents to the circle at A and B. We know that radius is perpendicular to the tangent at the point of contact or diameter is perpendicular to the tangent at the point of contact.




$\therefore$ $\angle 1 = 90°$ and $\angle 2 = 90°$ [$\because$ See from figure]

$\Rightarrow$ $\angle 1 = \angle 2$

But these are alternate angles.

$\therefore$ $l_1$ is parallel to $l_2$.

[$\because$ If alternate angles are equal, so 2 lines are parallel]

## Long Answer Type Questions [4 Marks]

**Question 35.**

Prove that the length of the tangents drawn from an external point to a circle are equal.

**Solution:**

Refer to Ans. 10.

**Question 36.**

Prove that a parallelogram circumscribing a circle is a rhombus

**Solution:**

**Given:** ABCD is parallelogram circumscribing a circle.

**To prove:** ABCD is a rhombus

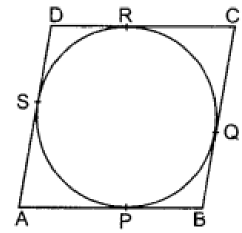
**Proof:** We have, $DR = DS$ ...(*i*)

[Lengths of tangents drawn from an external point to a circle are equal]

Also, $AP = AS$ ...(*ii*)

$BP = BQ$ ...(*iii*)

$CR = CQ$ ...(*iv*)




Adding (*i*), (*ii*), (*iii*) and (*iv*),

$$(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)$$

$\Rightarrow$ $CD + AB = AD + BC$

$\Rightarrow$ $2AB = 2AD$ [$\because$ In parallelogram, opposite sides are equal $AB = CD$ and $AD = BC$]

$\Rightarrow$ $AB = AD$

$\therefore$ $AB = AD = BC = CD$

Hence, ABCD is a rhombus as all sides are equal in rhombus.

**Question 37.**

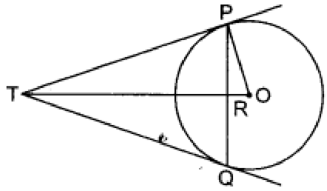
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

**Solution:**

Refer to Ans. 12.

**Question 38.**

In figure, PQ is a chord of length 16 cm, of a circle of radius 10 cm. The tangents at P and Q intersect at a point T. Find the length of TP.




**Solution:**

**Given:** PQ is chord of length 16 cm, TP and TQ are the tangents to a circle with centre O, radius 10 cm.

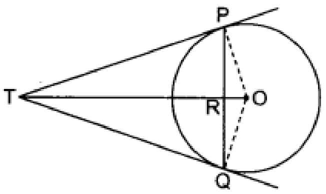
**To find:** TP.

**Solution:** Join OP and OQ.

In triangles OTP and OTQ,

OT is common

$OP = OQ$ (radii)



$$TP = TQ$$

[length of the tangents drawn from a point outside the circle to the circle are equal]

$\therefore \quad \Delta OPT \cong \Delta OQT$ (SSS congruence rule)

$\therefore \quad \angle POT = \angle QOT$ ...(*i*) (By CPCT)

Consider, triangles OPR and OQR

$OP = OQ$ (radii)

OR is common $\quad \angle POR = \angle QOR$ [from (*i*)]

$\therefore \quad \Delta OPR \cong \Delta OQR$ (SAS congruence rule)

So, $\quad PR = RQ = \frac{1}{2} \times 16 = 8$ cm ...(*ii*) (By CPCT)

$\angle ORP = \angle ORQ = 90°$ ...(*iii*) (By CPCT)

In right-angled triangle TRP,

$TR^2 = TP^2 - (8)^2 = TP^2 - 64$ ...(*iv*) [From (*iii*)]

Also, in $\Delta$TOP, $\quad OT^2 = TP^2 + (10)^2 = TP^2 + PO^2$ ($\because$ Pythagoras theorem)

$(TR + OR)^2 = TP + 100$

$(TR + 6)^2 = TP^2 + 100$ $\quad [\because OR = \sqrt{100-64} = 6]$

$TR^2 + 12TR + 36 = TP^2 + 100$

$TP^2 - 64 + 12TR + 36 = TP^2 + 100$ [From (*iv*)]

$12TR = 128 \Rightarrow TR = \frac{32}{3}$ cm

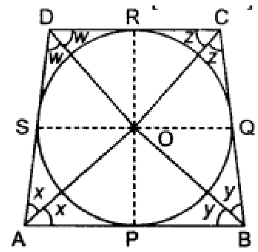
From (*iv*), $\quad \left(\frac{32}{3}\right)^2 = TP^2 - 64$

$\Rightarrow \quad TP^2 = \frac{1024}{9} + 64 = \frac{1024+576}{9} = \frac{1600}{9} \Rightarrow TP = \frac{40}{3}$ cm.

**Question 39.**

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Solution:**




**Given:** ABCD is a quadrilateral, circumscribing a circle with centre O and touches the quadrilateral at P, Q, R and S respectively.

**To Prove:** (*i*) $\angle AOB + \angle COD = 180°$

(*ii*) $\angle BOC + \angle AOD = 180°$

**Construction:** Join OP, OQ, OR and OS.

**Proof:** Consider, triangles APO and ASO,

$AP = AS$

[Lengths of the tangents drawn from a point outside the circle to the circle are equal]

$OS = OP$ (radii)

OA is common

$\therefore \quad \Delta APO \cong \angle ASO$ (SSS congruency rule)

$\therefore \quad \angle OAP = \angle OAS = x$ (say) (CPCT)

Similarly, $\quad \angle OBP = \angle OBQ = y$ (say)

$\angle OCQ = \angle OCR = z$ (say)

and $\qquad \angle ODR = \angle ODS = w$ (say)

We have, $\angle DAB + \angle ABC + \angle BCD + \angle CDA = 360°$

[$\because$ Angle sum property of quadrilateral]

$\Rightarrow \quad 2x + 2y + 2z + 2w = 360°$

$\Rightarrow \quad x + y + z + w = 180° \qquad$ ...(*i*)

Consider, $\angle AOB + \angle COD = [180° - x - y] + [180° - w - z]$

[Sum of angles of a triangle is 180°]

$= 360° - (x + y + z + w)$

$= 360° - 180° \qquad$ [using (*i*)]

$\therefore \quad \angle AOB + \angle COD = 180°$

Again consider, $\angle BOC + \angle AOD$

$= [180° - y - z] + [180° - x - w]$

[Sum of angles of a triangle is 180°]

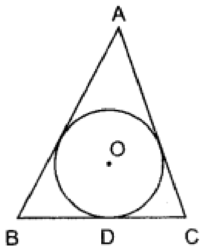
$= 360° - (x + y + z + w)$

$= 360° - 180° = 180° \qquad$ [using (*i*)]

Hence proved.

**Question 40.**

In figure, a triangle ABC is drawn to circumscribe a circle of radius 4 cm, such that the segments BD and DC are of lengths 8 cm and 6 cm respectively. Find the sides AB and AC.




**Solution:**

We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal in length.

$\therefore \quad AF = AE = x$, (say)

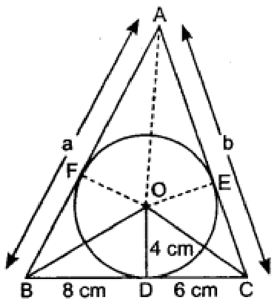
$BF = BD = 8$ cm

$CE = CD = 6$ cm

$\therefore \quad AB = (x + 8)$ cm $= a$ (say)

$AC = (x + 6)$ cm $= b$ (say)

and $\quad c$ (say) $= BC = 14$ cm $= (8 + 6)$ cm




Now, perimeter of triangle using Heron's formula

$2S = x + 8 + x + 6 + 14$

$= 28 + 2x$

$\Rightarrow \quad S = 14 + x$

$\therefore$ Area of $\Delta ABC$ using Heron's formula,

$= \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{(14+x)(14+x-x-8)(14+x-x-6)(14+x-14)}$

$= \sqrt{(14+x)\times 6\times 8\times x} \qquad$ ...(*i*)

Also, $\quad$ area of $\Delta ABC = ar(BOC) + ar(BOA) + ar(AOC)$

$= \frac{1}{2}\times 14\times 4 + \frac{1}{2}\times(8+x)\times 4 + \frac{1}{2}(6+x)\times 4$

$= 28 + 16 + 2x + 12 + 2x = 56 + 4x \qquad$ ...(*ii*)

From (*i*) and (*ii*), we get

$$\sqrt{48x(14+x)} = 56 + 4x$$

Squaring both sides, we get

$$48x(14 + x) = (56 + 4x)^2$$

$\Rightarrow$ $48x(14 + x) = 16(14 + x)^2$

$\Rightarrow$ $3x(14 + x) - (14 + x)^2 = 0$

$\Rightarrow$ $(14 + x)(3x - 14 - x) = 0$

$\Rightarrow$ $(14 + x)(2x - 14) = 0$

$\Rightarrow$ $14 + x = 0$ or $2x - 14 = 0$

$\Rightarrow$ $x = -14$ (rejected) or $x = 7$

$\therefore$ AB $= (7 + 8)$ cm $= 15$ cm,

and AC $= (7 + 6)$ cm $= 13$ cm.

**Question 41.**

Prove that the lengths of the tangents drawn from an external point to a circle are equal.

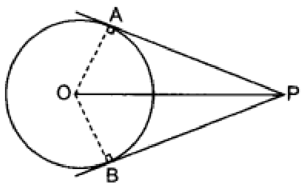
**Solution:**

**Given:** A circle $C(o, r)$. P is a point outside the circle and PA and PB are tangents to a circle.

**To Prove:** PA = PB

**Construct:** Draw OA, OB and OP.

**Proof:** Consider triangle OAP and OBP.




$\angle OAP = \angle OBP = 90°$ ...(*i*)

[Radius is perpendicular to the tangent at the point of contact]

OA = OB (radii) ...(*ii*)

OP is common ...(*iii*)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS) [from (*i*), (*ii*), (*iii*)]

$\Rightarrow$ AP = BP (CPCT)

**Question 42.**

A quadrilateral is drawn to circumscribe a circle. Prove that the sums of opposite sides are equal

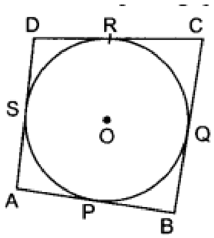
**Solution:**

**Given:** A quadrilateral ABCD which circumscribes a circle.

Let it touches the circle at P, Q, R and S as shown in figure.

**To Prove:** AB + CD = AD + BC

**Proof:** We know that the lengths of the tangents drawn from a point outside the circle to the circle are equal.




$\therefore$ AP = AS; BP = BQ; CQ = CR and DR = DS ...(*i*)

Consider, AB + CD = AP + PB + CR + RD

= AS + BQ + CQ + DS [using (*i*)]

= (AS + DS) + (BQ + CQ) = AD + BC

**2013**

**Short Answer Type Questions I [2 Marks]**

**Question 43.**

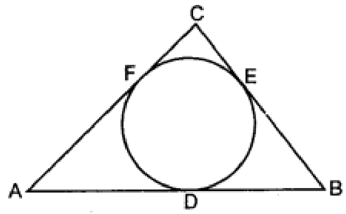
Prove that the parallelogram circumscribing a circle is a rhombus

**Solution:**

Refer to Ans. 36.

**Question 44.**

In the given figure, a circle inscribed in ΔABC touches its sides AB, BC and AC at points D, E and F respectively. If AB = 12 cm, BC = 8 cm and AC = 10 cm, then find the lengths of AD, BE and CF




**Solution:**

Given, AB = 12 cm, BC = 8 cm, AC = 10 cm

Let AD = $x$ cm

$\therefore$ BD = AB – AD = $(12 - x)$ cm

$\because$ AD = AF [tangents from point A]

$\therefore$ AF = $x$ cm

Now, CF = AC – AF = $(10 - x)$ cm

Also, CE = CF $\Rightarrow$ CE = $(10 - x)$ cm

And BD = BE [$\because$ tangents from B]

$\Rightarrow$ BE = $(12 - x)$ cm [From $(i)$]

Now, BC = CE + BE

$\Rightarrow$ $8 = (10 - x) + (12 - x)$

$\Rightarrow$ $8 = 22 - 2x \Rightarrow 2x = 14$

$\Rightarrow$ $x = 7$ cm

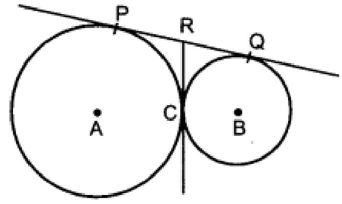
$\Rightarrow$ AD = 7 cm

BE = $12 - x = 12 - 7 = 5$ cm

and CF = $10 - x = 10 - 7 = 3$ cm

**Question 45.**

In the given figure, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.




**Solution:**

PR and RC are tangents to circle with centre A.

$\therefore$ PR = RC [tangent from common point R] ...$(i)$

Similarly RQ and RC are tangents to circle with centre B

$\therefore$ RQ = RC ...$(ii)$

From $(i)$ and $(ii)$, PR = RQ

$\therefore$ CR bisects PQ.

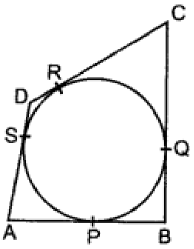
**Question 46.**

In the given figure, a quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC

**Solution:**

Quadrilateral ABCD circumscribing a circle.

$\therefore$ AP = AS [tangents drawn from common external point to a circle are equal in length.]




BP = BQ

DR = DS

CR = CQ

On adding,

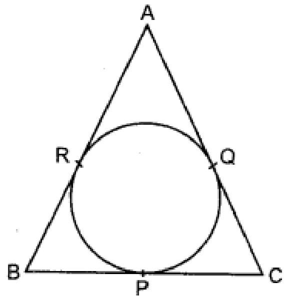
$$AP + BP + DR + CR = AS + BQ + DS + CQ$$

$$(AP + BP) + (DR + CR) = (AS + DS) + (BQ + CQ)$$

$$AB + CD = AD + BC$$

**Question 47.**

In the given figure, a circle inscribed in ΔABC, touches its sides BC, CA and AB at the points P, Q and R respectively. If AB = AC, then prove that BP = CP.




**Solution:**

$$AB = AC$$

$\therefore$ $AR + BR = AQ + CQ$

$AR + BR = AR + CQ$

[AQ = AR, euqal tangents]

$\Rightarrow$ $BR = CQ$

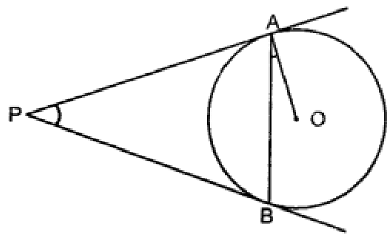
Now, $BR = BP$ [Length of equal tangents]

$CQ = CP$

$\Rightarrow$ $BP = CP$

**Question 48.**

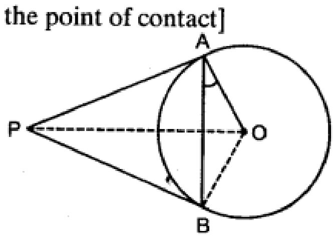
In the given figure, two tangents PA and PB are drawn to a circle with centre O from an external point P. Prove that ∠APB = 2 ∠OAB




**Solution:**

**Construction:** Join OP and OB.

**Proof:** Now, OA ⊥ AP [Radius is perpendicular to tangent at the point of contact]




$\Rightarrow$ $\angle OAP = 90°$

Similarly, $OB \perp BP$

$\Rightarrow$ $\angle OBP = 90°$

In quadrilateral OAPB

$$\angle OAP + \angle OBP = 90° + 90° = 180°$$

$\therefore$ Quadrilateral OAPB is a cyclic quadrilateral.

$\Rightarrow$ $\angle OAB = \angle OPB$ [Angles in same segment] ...(*i*)

Also, $\angle OPB = \angle OPA$ [OP bisects ∠APB]

$\Rightarrow$ $\angle OPB + \angle OPA = 2\angle OPB$ [From (*i*)]

$\Rightarrow$ $\angle APB = 2\angle OPB$ ...(*ii*)

From (*i*) and (*ii*)

$\angle APB = 2\angle OAB$ [$\because \angle OAB = \angle OPB$]

## Long Answer Type Questions [4 Marks]

**Question 49.**

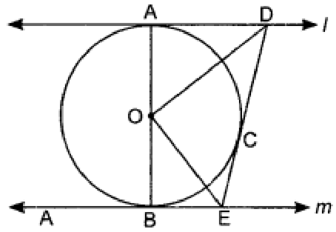
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

**Solution:**

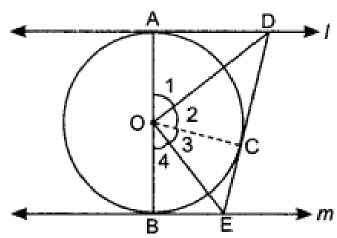
Refer to ANS.12

**Question 50.**

In the given figure l,m are two parallel tangents to the circle with center O,touching the circle at A and B respectively.Another tangent at C intersect the line l at D and m at E. prove that ∠DOE=90




**Solution:**




**Given:** Line $l \parallel m$ and both are tangents to a circle at points A and B respectively.

**To prove:** $\angle DOE = 90^\circ$

**Construction:** Join OC

**Proof:** In $\Delta ADO$ and $\Delta CDO$,

$AD = DC$ [tangents from an external point are equal]

$OD = OD$ [common]

$\angle OAD = \angle OCD = 90^\circ$ [radius is perpendicular to tangent]

$\therefore \Delta ADO \cong \Delta COD$ [By RHS]

$\therefore \angle 1 = \angle 2$ ...(*i*) [By CPCT]

Similarly, in $\Delta BOE$ and $\Delta COE$,

$\angle 3 = \angle 4$ ...(*ii*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 = 180^\circ$ [angles on a straight line]

$\Rightarrow 2\angle 2 + 2\angle 3 = 180^\circ$ [$\because \angle 1 = \angle 2$ and $\angle 3 = \angle 4$]

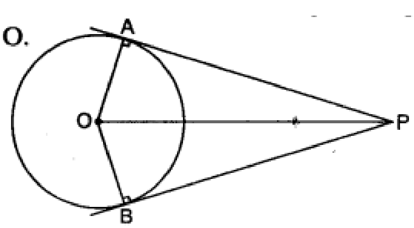
$\Rightarrow 2(\angle 2 + \angle 3) = 180^\circ$

$\Rightarrow \angle 2 + \angle 3 = 90^\circ \Rightarrow \angle DOE = 90^\circ$

**Question 51.**

Prove that the lengths of tangents drawn from an external point to a circle are equal.

**Solution:**




**Given:** PA and PB are tangents to a circle with centre O.

**To Prove:** PA = PB

**Construction:** Join OP, OA, OB.

**Proof:** In $\Delta AOP$ and $\Delta BOP$

$OP = OP$ (common)

$OA = OB$ (radii of circle)

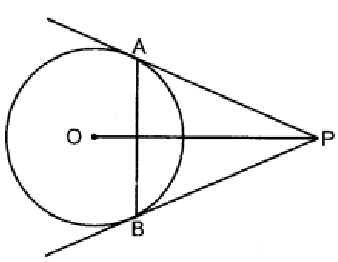
$\angle OAP = \angle OBP = 90^\circ$ (radius is perpendicular to tangent)

$\Delta AOP \cong \Delta BOP$ (RHS)

$\therefore PA = PB$ (CPCT)

**Question 52.**

In the given figure, PA and PB are two tangents drawn from an external point P to a circle with centre O. Prove that OP is the right bisector of line segment AB.



**Solution:**

Join OA and OB.

In $\Delta PAO$ and $\Delta PBO$

$OA = OB$ [Radii]

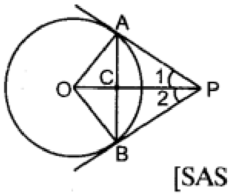
$OP = OP$ [Common]

and $AP = BP$ [Tangents from P]

$\therefore \Delta PAO \cong \Delta PBO$ (SSS congruence rule)

$\Rightarrow \angle 1 = \angle 2$

In $\Delta APC$ and $\Delta BPC$




$\angle 1 = \angle 2$ [Proved]

$AP = BP$

and $PC = PC$

$\therefore \Delta APC \cong \Delta BPC$ [SAS congruence rule]

$\Rightarrow AC = BC$ [CPCT]

and $\angle ACP = \angle BCP$

Also, $\angle ACP + \angle BCP = 180°$

$\Rightarrow \angle ACP = \angle BCP = 90°$

$\therefore$ OP is the right bisector of AB.

**Question 53.**

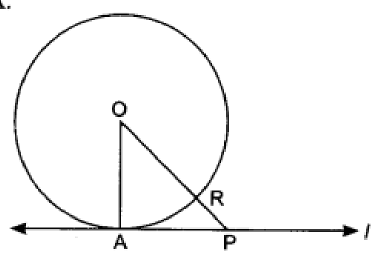
Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

**Solution:**

**Given:** A circle with centre O, line $l$ is tangent to the circle at A.

**To Prove:** Radius OA is perpendicular to the tangent at A.

**Construction:** Take a point P, other than A, on tangent $l$. Join OP, meeting the circle at R.




**Proof:** We know that tangent to the circle touches, the circle at one point and all other points on the tangent lie in the exterior of a circle.

$\therefore$ $OP > OR$ (radius of circle)

$\Rightarrow$ $OP > OA$ ($\because OR = OA$, radius of circle)

$\Rightarrow$ $OA < OP$

$\Rightarrow$ OA is the smallest segment, from O to a point on the tangent.

We know that smallest line segment from a point outside the circle to the line is perpendicular segment.

Hence, $OA \perp$ tangent $l$.

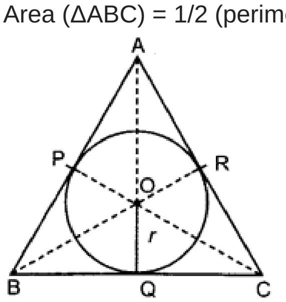
$\Rightarrow$ tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Question 54.**

In the given figure, the sides AB, BC and CA of $\Delta ABC$ touch a circle with centre O and radius r at P, Q and R respectively.

Prove that:

1. AB + CQ = AC + BQ
2. Area ($\Delta ABC$) = 1/2 (perimeter of $\Delta ABC$) X r



**Solution:**

(i) We have, $AP = AR$ [Tangents from A] ...(i)

Similarly, $BP = BQ$ [Tangents from B] ...(ii)

$CR = CQ$ [Tangents from C] ...(iii)

Now, we have

$$\because \quad AP = AR$$

$$\Rightarrow \quad (AB - BP) = (AC - CR)$$

$$\Rightarrow \quad AB + CR = AC + BP$$

$$\Rightarrow \quad AB + CQ = AC + BQ \quad \text{[Using eq. (ii) and (iii)]}$$

(ii) $$\text{Area}(\Delta ABC) = \text{Area}(\Delta ABO + \Delta OBC + OAC)$$

$$= \frac{1}{2}(AB + BC + AC) \times r \quad [\therefore \text{ Area }(\Delta) = \frac{1}{2} \times \text{base} \times \text{height}]$$

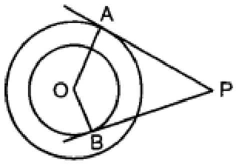
$$= \frac{1}{2}(\text{perimeter of } \Delta ABC) \times r$$

## 2012

### Short Answer Type Questions I [2 Marks]

**Question 55.**

Tangents PA and PB are drawn from an external point P to two concentric circles with centre O and radii 8 cm and 5 cm respectively, as shown in Fig. If AP = 15 cm, then find the length of BP



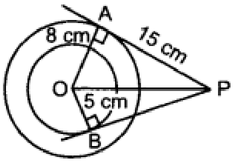

**Solution:**

Join OP.

In $\Delta PAO$ and $\Delta PBO$

$$\angle PAO = 90^\circ, \angle PBO = 90^\circ$$

($\because$ tangent is perpendicular to radius at the point of contact)




In right angled $\Delta PAO$

$$PA^2 + OA^2 = OP^2 \quad [\because \text{ Pythagoras theorem}]$$

$$15^2 + 8^2 = OP^2$$

$$225 + 64 = OP^2$$

$$OP^2 = 289$$

$$OP = \sqrt{289} = 17 \text{ cm}$$

Now, In right angled $\Delta PBO$

$$PB^2 + BO^2 = PO^2 \quad [\because \text{ Pythagoras theorem}]$$

$$PB^2 + 5^2 = (17)^2$$

$$PB^2 + 25 = 289$$

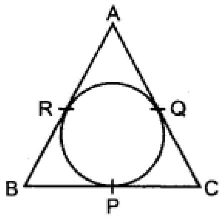
$$PB^2 = 289 - 25$$

$$PB^2 = 264$$

$$PB = \sqrt{264} = 2\sqrt{66} \text{ cm.}$$

**Question 56.**

In figure, an isosceles triangle ABC, with AB = AC, circumscribes a circle. Prove that the point of contact P bisects the base BC

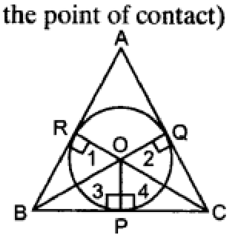


**Solution:**

Let the centre of circle be O.
Join OR, OQ, OB, OP, OC.

$$\angle 1 = \angle 2 = \angle 3 = \angle 4 = 90^\circ$$

(∵ Radius is perpendicular to tangent at the point of contact)




In $\Delta ORB$ and $\Delta OQC$

$OR = OQ$ (Radii of same circle)

$\angle 1 = \angle 2$ (each 90°)

$RB = QC$ $\left(\begin{array}{l}\because AB = AC \text{ and } AR = AQ \\ \text{So, } AB - AR = AC - AQ\end{array}\right)$

By SAS congruence rule,

$$\Delta ORB \cong \angle OQC$$

∴ $OB = OC$ (By CPCT)

In $\Delta OPB$ and $\Delta OPC$

$OP = OP$ (common)

$\angle 3 = \angle 4$ (each 90°)

$OB = OC$ (Proved above)

By RHS congruence rule,

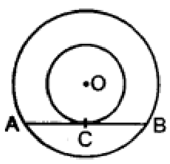
$$\Delta OPB \cong \Delta OPC$$

$BP = PC$ (By CPCT)

Hence, P bisects the base BC.

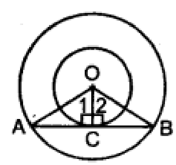
**Question 57.**

In figure, the chord AB of the larger of the two concentric circles, with centre O, touches the smaller circle at C. Prove that AC = CB.




**Solution:**

**Given:** Two concentric circles with centre O.
AB is chord of bigger circle which touches the smaller circle to C.
**To Prove:** AC = CB
**Construction:** Join OA, OC, OB
**Proof:** In $\Delta OCA$ and $\Delta OCB$




$OC = OC$ (common)

$\angle 1 = \angle 2$ (Radius perpendicular tangent)

$OA = OB$ (radii of same circle).

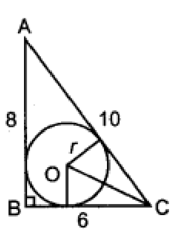
By RHS congruence rule,

$$\Delta OCA \cong \Delta OCB$$

∴ $AC = BC$

**Question 58.**

In figure, a right triangle ABC, circumscribes a circle of radius r. If AB and BC are of lengths 8 cm and 6 cm respectively, find the value of r.

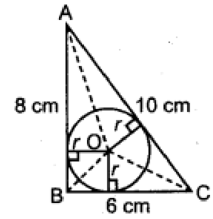


**Solution:**

$\because$ ABC is right angle $\Delta$, right $\angle d$ at B.

So, By Pythagoras theorem

$$AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 100$$
$$AC = 10 \text{ cm}$$




So, $\quad ar(\Delta ABC) = \frac{1}{2} \times 6 \times 8 = 24 \text{ cm}^2$

Also, $\quad ar(\Delta ABC) = ar(\Delta OBC) + ar(\Delta OAC) + ar(\Delta OAB)$

$\Rightarrow \quad 24 = \frac{1}{2} \times 6 \times r + \frac{1}{2} \times 10 \times r + \frac{1}{2} \times 8 \times r$

$\Rightarrow \quad 24 = 3r + 5r + 4r \quad \Rightarrow \quad 12r = 24$

$\Rightarrow \quad r = 2 \text{ cm}$

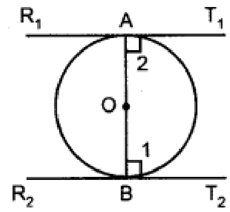
**Question 59.**

Prove that the tangents drawn at the ends of a diameter of a circle are parallel

**Solution:**

AB is the diameter.

$R_1T_1$ and $R_2T_2$ are the tangents at point A and B respectively.




Now, $\quad OB \perp R_2T_2$

[radius perpendicular the tangent at point of contact]

$\Rightarrow \quad \angle 1 = 90°$

Also, $\quad OA \perp R_1T_1$

[radius perpendicular the tangent at point of contact]

$\Rightarrow \quad \angle 2 = 90°$

Now, $\angle 1 + \angle 2 = 90° + 90° = 180°$

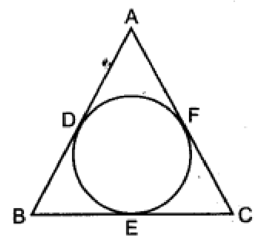
$\Rightarrow \quad R_1T_1 \parallel R_2T_2$ [$\because$ if interior angles on same side is supplementary, 2 lines are parallel]

**Question 60.**

The incircle of an isosceles triangle ABC, with AB = AC, touches the sides AB, BC and CA at D, E and F respectively. Prove that E bisects BC

**Solution:**

We have, $\quad AB = AC$ [Given] ...(*i*)




$AD = AF$

[tangents drawn from an external point are equal] ...(*ii*)

On subtracting eq (*ii*) from eq (*i*), we get

$AB - AD = AC - AF \Rightarrow BD = CF$ ...(*iii*)

$\because$ Also, $\quad BD = BE$ ...(*iv*)

$\therefore$ from (*iii*) and (*iv*), we have

$BE = CF$ ...(*v*)

Also, $\quad CF = CE$ ...(*vi*)

$\therefore$ from (*v*) and (*vi*)

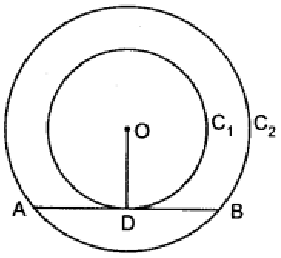
$BE = CE$

$\therefore$ E bisects BC.

**Question 61.**

Prove that in two concentric circles, the chord of the larger circle, which touches the smaller circle, is bisected at the point of contact

**Solution:**




**Given:** Let O be the centre of two concentric circles $C_1$ and $C_2$. Let AB be the chord of larger circle $C_2$ which is a tangent to the smaller circle $C_1$ at D.

**To prove:** Now we have to prove that the chord AB is bisected at D that is AD = BD.

**Construction:** Join OD.

**Proof:** Now since OD is the radius of the circle $C_1$ and AB is the tangent to the circle $C_1$ at D.

So, $OD \perp AB$ [radius of the circle is perpendicular to tangent at any point of contact]

Since AB is the chord of the circle $C_2$ and $OD \perp AB$.

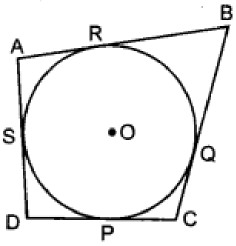
$\therefore$ AD = DB [perpendicular drawn from the centre to the chord always bisects the chord]

### Short Answer Type Questions II [3 Marks]

**Question 62.**

Prove that the parallelogram circumscribing a circle is a rhombus.

**Solution:**




**Given:** A circle with centre O.

ABCD is a parallelogram circumscribing the circle, touching it at P, Q, R, S

**To Prove:** ABCD is a Rhombus.

**Proof:** $AR = AS$ ...(*i*)

[$\because$ tangents from an external point are equal]

$RB = BQ$ ...(*ii*)

$DP = DS$ ...(*iii*)

$PC = CQ$ ...(*iv*)

Consider $AB + DC = AR + RB + DP + PC$

$= AS + BQ + DS + QC$ [By (*i*), (*ii*), (*iii*), (*iv*)]

Now, $AB + DC = AD + BC$

$AB + AB = AD + AD$

[$\because$ ABCD is a parallelogram, so opposite sides are equal, i.e. AB = CD. AD = BC]

$2AB = 2AD$

$AB = AD$

So, $AB = BC = CD = AD$

ABCD is a parallelogram with all sides equal.

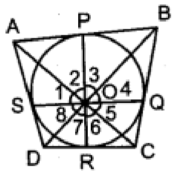
Hence, ABCD is a Rhombus.

**Question 63.**

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Solution:**

**Given:** ABCD is a quadrilateral circumscribing the circle with centre O touching it at P, Q, R, S.




**To Prove:** $\angle AOB + \angle DOC = 180°$

$\angle AOD + \angle BOC = 180°$

**Construction:** Join AO, PO, BO, QO, CO, RO, DO, SO,

**Proof:** In $\Delta AOS$ and $\Delta AOP$

$AO = AO$ (common)

$AS = AP$ (tangents from external point)

$OS = OP$ (radii of same circle)

By SSS congruence

$\Delta AOS \cong \Delta AOP$

$\angle 1 = \angle 2$ (By CPCT) ...(*i*)

Similarily, $\angle 3 = \angle 4, \angle 5 = \angle 6, \angle 7 = \angle 8$ ...(*ii*)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [$\because$ ASP of quadrilateral]

$\angle 2 + \angle 2 + \angle 3 + \angle 3 + \angle 6 + \angle 6 + \angle 7 + \angle 7 = 360°$ [By (*i*), (*ii*)]

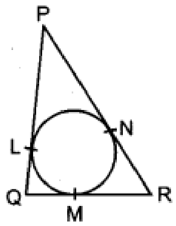
$2[\angle 2 + \angle 3 + \angle 6 + \angle 7] = 360°$

$\angle AOB + \angle COD = 180°$

Similarily, $\angle AOD + \angle BOC = 180°$

**Question 64.**

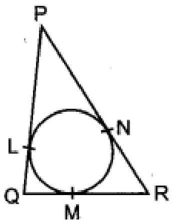
In figure, a circle is inscribed in a triangle PQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find the lengths QM, RN and PL.




**Solution:**

We know that the tangents drawn from an external point to a circle are equal.

Therefore




Let $QM = x = QL$

$MR = y = RN$

and $PL = z = PN$

Now $PQ = 10$ cm, $QR = 8$ cm, $PR = 12$ cm

$\Rightarrow$ $x + y = 8, y + z = 12, z + x = 10$

$\Rightarrow$ $2x + 2y + 2z = 8 + 12 + 10 = 30$

$\Rightarrow$ $x + y + z = 15 \Rightarrow 8 + z = 15 \Rightarrow z = 7$

$\Rightarrow$ $x + 12 = 15 \Rightarrow x = 3$

$\Rightarrow$ $y + 10 = 15 \Rightarrow y = 5$

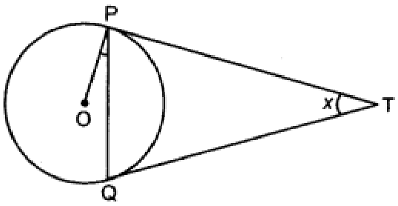
Hence, QM = 3 cm, RN = 5 cm and PL = 7 cm.

**Question 65.**

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$

**Solution:**

**Given:** A circle with centre O. External point T and two tangents TP and TQ to the circle, where P, Q are the points of contact.




**To Prove:** $\angle PTQ = 2\angle OPQ$

**Proof:** Let $\angle PTQ = x$

In $\Delta PTQ$, $PT = PQ$ [The lengths of tangents drawn from an external point to a circle are equal]

$\angle TPQ = \angle TQP$ [angles opposite to equal sides are equal]

$\angle TPQ = \angle TQP = \frac{1}{2}(180° - x) = 90° = \frac{x}{2}$ [$\because$ ASP of triangle]

$\angle OPT = 90°$ [The tangent at any point of a circle is perpendicular to the radius through the point of contact]

$\therefore$ From figure, $\angle OPQ = \angle OPT - \angle TPQ$

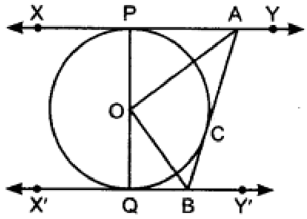
$$= 90° - \left(90° - \frac{x}{2}\right) = 90° - 90° + \frac{x}{2}$$

$$\angle OPQ = \frac{1}{2}\angle PTQ$$

This gives $\angle PTQ = 2\angle OPQ$ Hence, Proved.

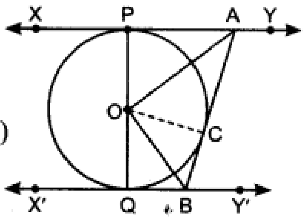
**Question 66.**

In figure, XY and X'Y' are two parallel tangents to a circle with centre O and another tangent AB with point of contact C intersects XY at A and X'Y' at B. Prove that $\angle AOB = 90°$.




**Solution:**

**Given:** XY and X'Y' are are two parallel tangents to circle with centre O. Tangent AB with point of contact C intersects XY at A and X'Y' at B.




**To Prove:** $\angle AOB = 90°$

**Construction:** Join OC.

**Proof:** In $\Delta OPA$ and $\Delta OCA$,

$OP = OC$ (Radii of the same circle)

$AP = AC$ (Tangents from point A)

$AO = AO$ (common side)

$\Delta OPA \cong \Delta OCA$ (SSS congruence rule)

Therefore, $P \to e, A \to A, O \to o$,

$\angle POA = \angle COA$ ...(*i*) (CPCT)

Similarly, we prove: $\Delta OQB \cong \Delta OCB$

Then: $\angle QOB = \angle COB$ ...(*ii*) (CPCT)

Since, POQ is the diameter of the circle, it is a straight line.

$\therefore$ $\angle POA + \angle COA + \angle COB + \angle QOB = 180°$

from equation (*i*) and (*ii*),

$$2\angle COA + 2\angle COB = 180°$$

$$2(\angle COA + \angle COB) = 180°$$

$$\angle COA + \angle COB = \frac{180°}{2}$$

$$\angle COA + \angle COB = 90°$$

$$\angle AOB = 90°$$

Hence, Proved.

**Question 67.**

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

**Solution:**

Refer to Ans. 12.

**Question 68.**

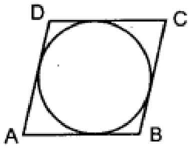
A quadrilateral ABCD is drawn to circumscribe a circle. Prove that AB + CD = AD + BC.

**Solution:**

Refer to Ans. 46.

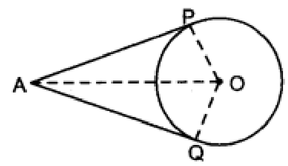
**Question 69.**

Prove that the lengths of tangents drawn from an external point to a circle are equal. Using it, prove: quadrilateral ABCD is drawn to circumscribe a circle. Such'that AB + CD = AD + BC




**Solution:**

**Given:** A circle with centre O. Through the external point A, tangents AP and AQ are drawn.




**To prove:** AP = AQ

**Construction:** Join OA, OP and OQ

**Proof:** In $\Delta OAP$ and $\Delta OAQ$,

$OP = OQ$ [Radii of the same circle]

$OA = OA$ [Common]

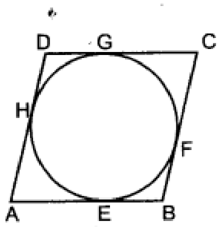
$\angle OPA = \angle OQA = 90^\circ$ [radius is perpendicular to the tangent at point of contact]

$\therefore$ $\Delta OAP \cong \Delta OAQ$ [By RHS]

$\Rightarrow$ $AP = AQ$ [CPCT]

Hence proved.

**Second Part:**




In the given figure, $AE = AH$ ...(*i*)

[Tangents drawn from an external point are equal]

$BE = BF$ ...(*ii*)

[Tangents drawn from an external point are equal]

$DG = DH$ ...(*iii*)

[Tangents drawn from an external point are equal]

$CG = CF$ ...(*iv*) [Tangents drawn from an external point are equal]

Adding equation (*i*), (*ii*), (*iii*) and (*iv*), we get

$$AE + BE + DG + CG = AH + BF + DH + CF$$

$\Rightarrow$ $(AE + BE) + (DG + CG) = (AH + DH) + (BF + CF)$

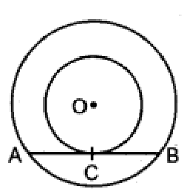
$\Rightarrow$ $AB + CD = AD + BC$.

Hence proved.

**Question 70.**

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact.

**Solution:**



Refer to Ans. 12.

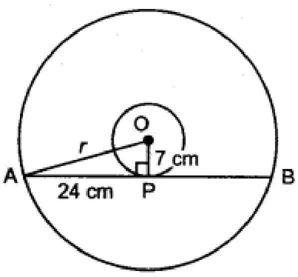
## 2011

### Short Answer Type Questions I [2 Marks]

**Question 71.**

Two concentric circles are of radii 7 cm and r cm respectively, where r >7 .A chord of the larger circle, of length 48 cm, touches the smaller circle. Find the value of r.

**Solution:**




**Given:** $OP = 7$ cm; $OA = r$ cm

$AB = 48$ cm

Now, $OP \perp AB$ (as radius makes an angle of 90° with the tangent at point of contact)

Also, $AP = PB$ (perpendicular drawn from centre to the chord bisects the chord)

So, $AP = 24$ cm

In $\Delta OPA$, $\angle P = 90°$

By Pythagoras theorem in $\Delta OPA$,

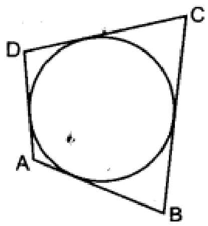
$$OA^2 = AP^2 + OP^2$$

$$r^2 = 24^2 + 7^2 = 576 + 49 = 625$$

$\Rightarrow$ $r = 25$ cm

**Question 72.**

In figure, a circle touches all the four sides of a quadrilateral ABCD whose sides are AB = 6 cm, BC = 9 cm and CD = 8 cm. Find the length of side AD.

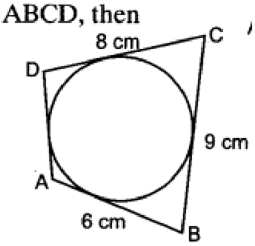



**Solution:**

If a circle touches all the four sides of quadrilateral ABCD, then we know that $AD + BC = AB + CD$

$\therefore$ $AD + 9 = 6 + 8$

$\Rightarrow$ $AD = 5$ cm

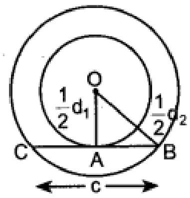



**Question 73.**

If d1, d2 (d2 > d1) be the diameters of two concentric circles and c be the length of a chord of a circle which is tangent to the other circle, prove that $d^{2}2 = c^2 + d^2$.

**Solution:**

$\because$ Diameter of bigger circle $= d_2$




So, Radius of bigger circle $= \frac{1}{2}d_2 = OB$

and Diameter of smaller circle $= d_1$

So, Radius of smaller circle $= \frac{1}{2}d_1 = OA$

$$AB = \frac{c}{2} \quad [\because \text{ Diameter of CB} = \text{'C'}]$$

In right $\Delta OAB$, $\angle A = 90°$

[$\because$ radius is perpendicular the tangent at point of contact]

By pythagoras theorem $OB^2 = AB^2 + OA^2$

$$\Rightarrow \left(\frac{1}{2}d_2\right)^2 = \left(\frac{1}{2}c\right)^2 + \left(\frac{1}{2}d_1\right)^2 \Rightarrow \frac{1}{4}d_2^2 = \frac{1}{4}c^2 + \frac{1}{4}d_1^2$$

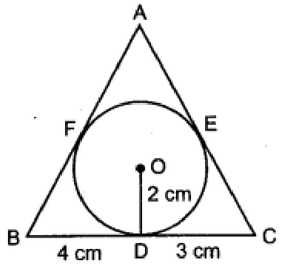
$$\Rightarrow d_2^2 = c^2 + d_1^2$$

Hence proved.

## Short Answer Type Questions II [3 Marks]

### Question 74.

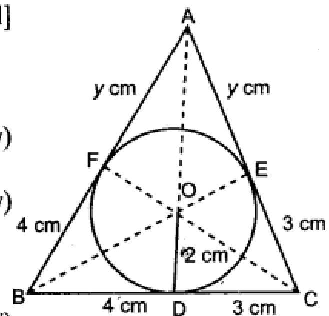
**In figure, a triangle ABC is drawn to circumscribe a circle of radius 2 cm such that the segments BD and DC into which BC is divided by the point of contact D are the lengths 4 cm and 3 cm respectively. If area of $\Delta ABC = 21$ cm$^2$, then find the lengths of sides AB and AC.**




### Solution:

Let $AE = AF = y$ (say)

[Tangents drawn from an external point are equal]




$$\text{ar } \Delta BOC = \frac{1}{2} \times 7 \times 2 = 7 \text{ cm}^2 = b \text{ (say)}$$

$$\text{ar } \Delta AOB = \frac{1}{2} \times (4 + y) \times 2 = (4 + y) \text{ cm}^2 = a \text{ (say)}$$

$$\text{ar } \Delta AOC = \frac{1}{2} \times (3 + y) \times 2 = (3 + y) \text{ cm}^2 = c \text{ (say)}$$

Now, ar $\Delta ABC$ = ar $\Delta AOB$ + ar $\Delta BOC$ + ar $\Delta AOC$

$= 4 + y + 7 + 3 + y$

ar $\Delta ABC = 14 + 2y$ ...(*i*)

For $\Delta ABC$,

Semi-perimeter, $$s = \frac{a+b+c}{2} = \frac{4+y+7+3+y}{2} = \frac{14+2y}{2} = 7 + y$$

$\therefore$ $$\text{ar }\Delta ABC = \sqrt{s(s-a)(s-b)(s-c)} \quad [\because \text{ By Heron's formula}]$$
$$= \sqrt{(7+y)(7+y-4-y)[7+y-7)(7+y-3-y)}$$
$$= \sqrt{(7+y)\times 3\times y\times 4}$$
$$\text{ar }\Delta ABC = 2\sqrt{3y(7+y)} \quad ...(ii)$$

From (*i*) and (*ii*)

$$\Rightarrow \quad 2\sqrt{3y(7+y)} = 14 + 2y$$
$$\Rightarrow \quad \sqrt{3y(7+y)} = 7 + y$$

Squaring both sides, we get

$$\Rightarrow \quad 3y(7+y) = (7+y)^2 \Rightarrow 21y + 3y^2 = 49 + y^2 + 14y$$
$$\Rightarrow \quad 2y^2 + 7y - 49 = 0 \Rightarrow 2y^2 + 14y - 7y - 49 = 0$$
$$\Rightarrow \quad 2y(y+7) - 7(y+7) = 0 \Rightarrow (2y-7)(y+7) = 0$$
$$\Rightarrow \quad y = \frac{7}{2}, y = -7 \quad \text{[Rejected]}$$

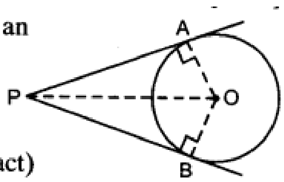
Hence, length of side AB = 4 + 3.5 = 7.5 cm and AC = 3 + 3.5 = 6.5 cm.

## Long Answer Type Questions [4 Marks]

**Question 75.**

Prove that the lengths of tangents drawn from an external point to a circle are equal.

**Solution:**




**Given:** PA and PB are two tangents to a given circle drawn from an external point P.

**To prove:** PA = PB

**Proof:** OA $\perp$ PA and OB $\perp$ PB

(radius perpendicular to tangent at point of contact)

Join OP.

Now, In $\Delta OAP$ and $\Delta OBP$,

| | | |
|---|---|---|
| | OA = OB | (radii) |
| | $\angle A = \angle B$ | (each 90°) |
| | OP = OP | (common) |
| So, | $\Delta OAP \cong \Delta OBP$ | (By RHS) |
| So, | PA = PB | (By CPCT) |

**Question 76.**

Prove that the tangent at any point of a circle is perpendicular to the radius through the point of contact

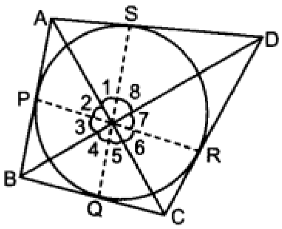
**Solution:**

Refer to Ans. 12.

**Question 77.**

Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

**Solution:**




The given quadrilateral ABCD is circumscribing the circle having its centre at O.

The sides AB, BC, CD and AD touch the circle at P, Q, R and S respectively.

Join OA, OB, OC, OD; OP, OQ, OR, OS.

We observe that OA bisects $\angle POS$ [$\because$ By CPCT, applied to $\Delta POA$ and $\Delta SOA$]

$\Rightarrow \quad \angle 1 = \angle 2 \quad ...(i)$

similarly $\quad \angle 3 = \angle 4 \quad ...(ii)$

$\angle 5 = \angle 6 \quad ...(iii)$

and $\quad \angle 7 = \angle 8 \quad ...(iv)$

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$ [$\because$ ASP of quadrilateral]

$\Rightarrow \quad 2(\angle 1 + \angle 4 + \angle 5 + \angle 8) = 360°$

$\Rightarrow \quad (\angle 1 + \angle 8) + (\angle 4 + \angle 5) = 180° \Rightarrow \angle AOD + \angle BOC = 180°$

Similarly, $\quad \angle AOB + \angle COD = 180°$

Hence, opposite sides of the quadrilateral ABCD subtend supplementary angles at the centre

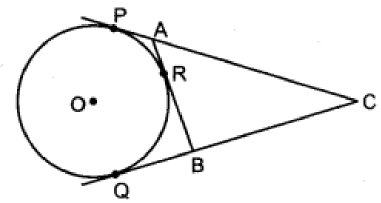
## 2010

### Very Short Answer Type Questions [1 Mark]

**Question 78.**

In figure, CP and CQ are tangents from an external point C to a circle with centre O. AB is another tangent which touches the circle at R. If CP = 11 cm and BR = 4 cm, find the length of BC.

**Solution:**




In the given figure, $\quad CP = CQ$

[tangents drawn from an external point are equal]

So, $\quad CP = CQ = 11$ cm

Also, $\quad BR = BQ$

[tangents drawn from an external point are equal]

So, $\quad BR = BQ = 4$ cm

$\therefore$ Now, $\quad BC = CQ - BQ = (11 - 4)$ cm $= 7$ cm

**Question 79.**

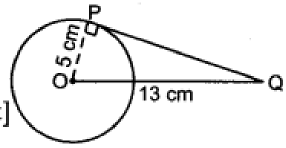
A tangent PQ at a point P of a circle of radius 5 cm meets a line through the centre O at a point Q so that OQ = 13 cm. Find the length PQ.

**Solution:**

Given, $\quad OP = 5$ cm [radius]

$OQ = 13$ cm

Now, In $\Delta OPQ$, $\angle P = 90°$ [radius is perpendicular to tangent at point of to contact]




$(OQ)^2 = (OP)^2 + (PQ)^2$

$\therefore \quad PQ = \sqrt{(13)^2 - (5)^2} = 12$ cm [By pythagoras theorem]

### Short Answer Type Questions I [2 Marks]

**Question 80.**

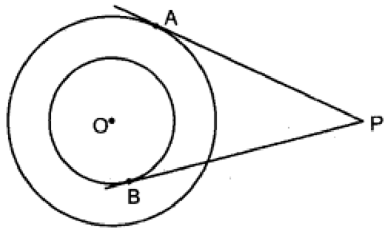
Prove that the lengths of tangents drawn from an external point to a circle are equal. Using the above prove the following: In Fig., PA and PB are tangents from an external point P , to a circle with centre O. LN touches the circle at M. Prove that PL + LM = PN + MN.

**Solution:**

Refer to Ans. 10 and 33.

**Question 81.**

In figure, there are two concentric circles, with centre O and of radii 5 cm and 3 cm. From an external point P, tangents PA and PB are drawn to these circles. If AP = 12 cm, find the length of BP.



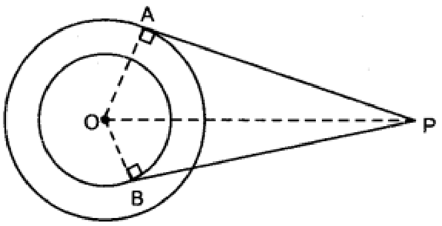

**Solution:**

**Construction:** Join OA, OB and OP.

AP = 12 cm, OA = 5 cm, OB = 3 cm

In $\Delta AOP$, $\angle A = 90°$ [radius is perpendicular to the tangent at point of contact]

$\Delta BOP$, $\angle B = 90°$ [radius is perpendicular to the tangent at point of contact]




So, $OP^2 = OA^2 + AP^2$

and $OP^2 = OB^2 + BP^2$

Using Pythagoras theorem for $\Delta AOP$ and $\Delta BOP$.

$$\therefore \quad OA^2 + AP^2 = OB^2 + BP^2$$

$$5^2 + 12^2 = 3^2 + BP^2 \Rightarrow 25 + 144 = 9 + BP^2 \Rightarrow 169 - 9 = BP^2$$

$$\Rightarrow \quad BP = \sqrt{160} \text{ cm} = 12.65 \text{ cm}$$